OC Tutorial – Semiconductor Basics

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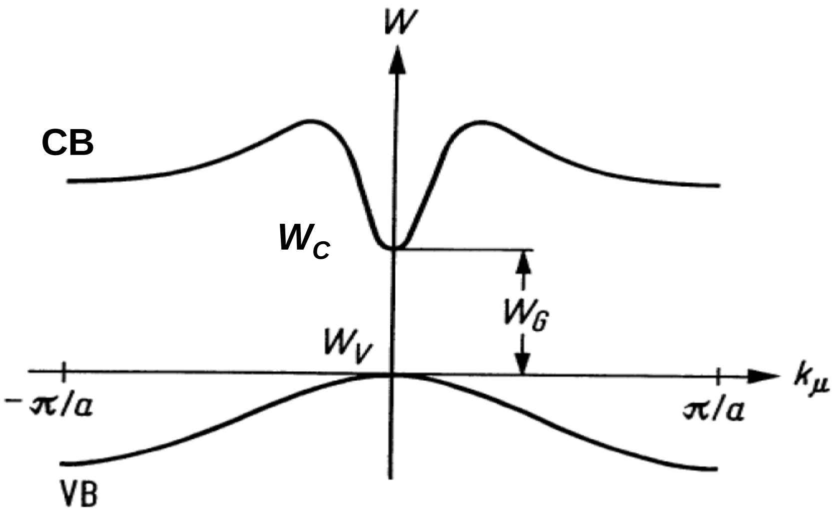
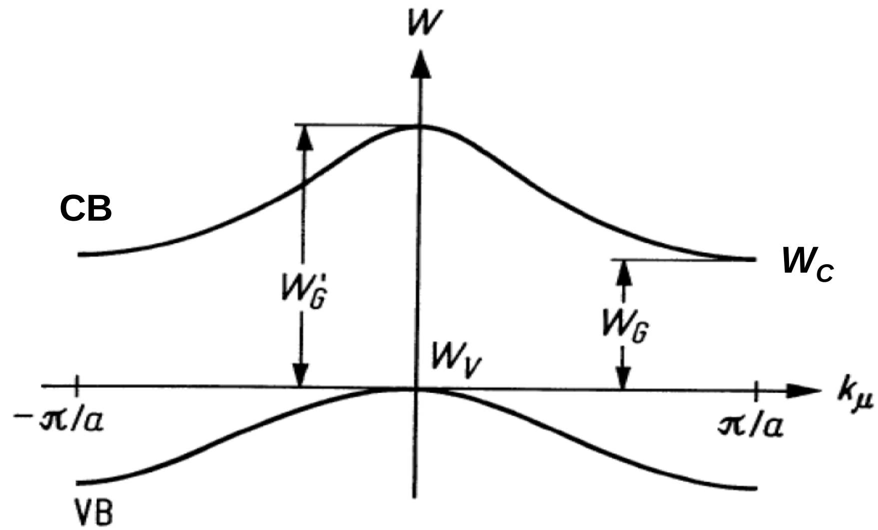
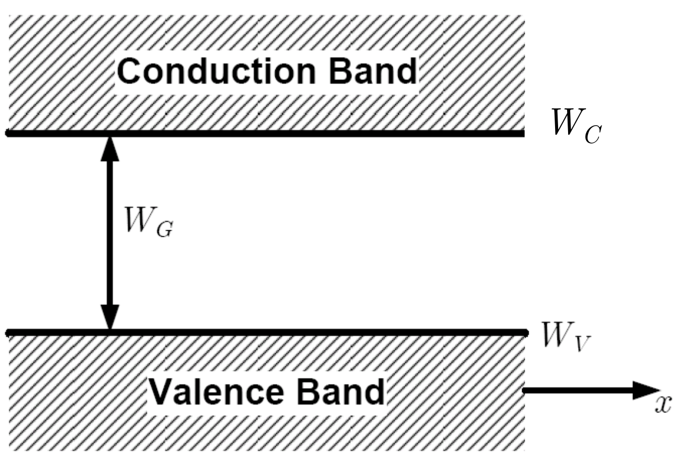
Direct and Indirect Semiconductors

Direct semiconductor (e.g. GaAs, InP)



Indirect semiconductor (e.g. Si, Ge)





Direct semiconductor

Maximum of valence band and minimum of conduction band at same k_μ.

Indirect semiconductor

Maximum of valence band and minimum of conduction band at different k_μ.

Intrinsic and Extrinsic Semiconductor

Intrinsic semiconductor: Pure semiconductor with negligible amount of impurities. Electron and hole carrier concentrations in thermal equilibrium are determined by material properties and temperature:

$$n_T = p = n_i$$

Extrinsic semiconductor: Doping changes carrier concentrations in thermal equilibrium. *Donors* "donate" negatively charged electrons to the conduction band (n-type). *Acceptors* "accept" additional electrons, and positively charged "holes" are created in the valence band (p-type).

n-type:

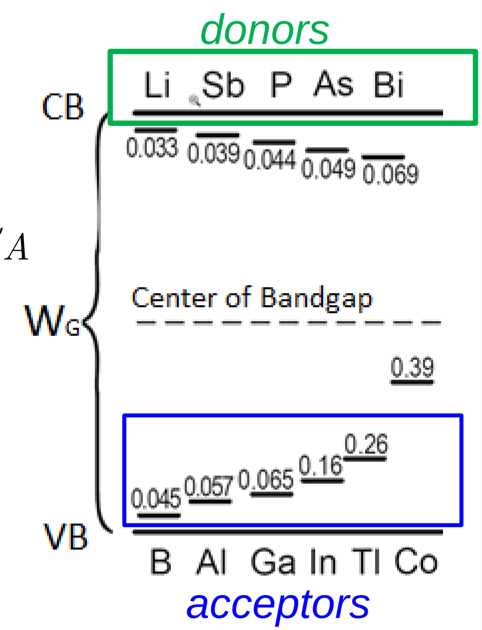
majorities: $n_T \cong n_D$

minorities: $p_{n0} = n_i^2 / n_D$

p-type:

majorities: $p \cong n_A$

minorities: $n_{p0} = n_i^2 / n_A$



Typical doping concentrations: 10^{15} cm^{-3} to 10^{18} cm^{-3}

Intrinsic *and* extrinsic semiconductor in thermal equilibrium (*mass-action law*): $n_T p = n_i^2$

Neutrality condition: $n_T + n_A = p + n_D$

Carrier Concentration at Thermal Equilibrium

Density of states in the conduction band (ρ_C , number of electron states per energy interval), and in the valence band (ρ_V, number of hole states per energy interval):

$$\rho_C(W) = \frac{1}{2\pi^2}\left(\frac{2\left|m_n\right|}{\hbar^2}\right)\sqrt{W - W_C} \qquad \rho_V(W) = \frac{1}{2\pi^2}\left(\frac{2\left|m_p\right|}{\hbar^2}\right)\sqrt{W_V - W}$$

Carrier concentration in conduction band (n_T) and valence band (p):

$$n_T = \int_{W_C}^{\infty} \rho_C(W) f(W) \mathrm{d}W \qquad p = \int_{-\infty}^{W_V} \rho_V(W)\left[1 - f(W)\right] \mathrm{d}W$$

$f(W)$ is the *Fermi-Dirac distribution function*. $f(W)$ is the probability that a state at energy W is occupied by an electron.

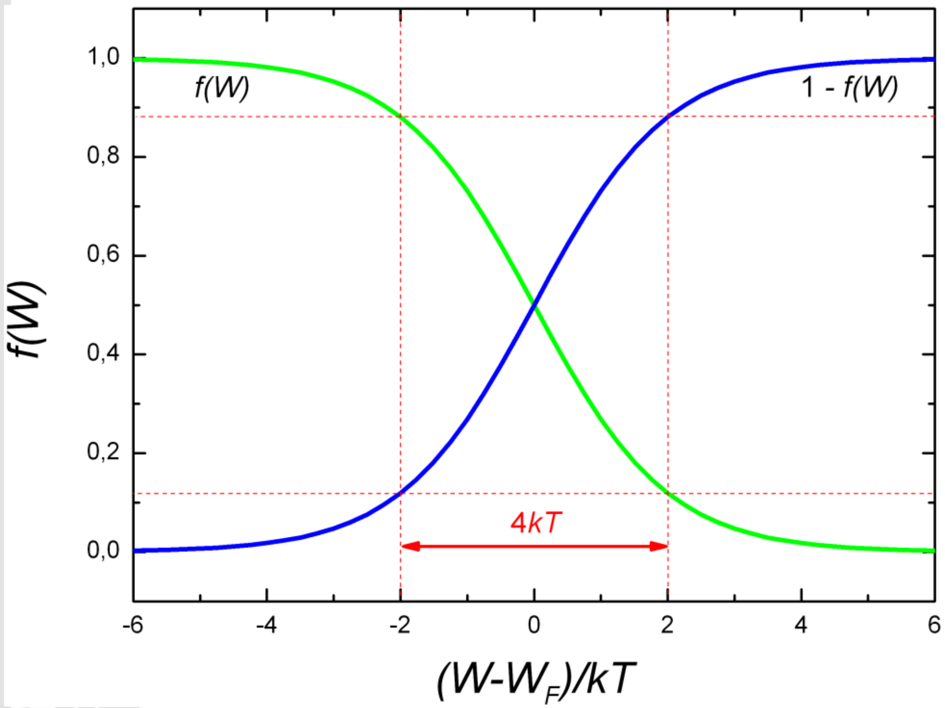
$1 - f(W)$ is the probability that a state at energy W is not occupied by an electron, i. e., that it is occupied by a hole.

Fermi-Dirac Distribution Function

$$f(W) = \frac{1}{1 + \exp\left(\frac{W - W_F}{kT}\right)}$$

$$1 - f(W) = \frac{1}{1 + \exp\left(\frac{W_F - W}{kT}\right)}$$

k	Boltzmann's constant
kT	Thermal energy $kT = 25\text{meV}$ at $T = 293\text{K}$
W_F	Fermi energy



- At Fermi energy, $f(W_F) = 0.5$
- Position of Fermi level:
  - Intrinsic: Between W_V and W_C
  - n-type: W_F moves towards W_C
  - p-type: W_F moves towards W_V
- Transition region:
  $(0.88 > f > 0.12) \rightarrow$ width $4kT$
  $\Delta f = 4kT/h = 24.2\ \text{THz}$

Boltzmann Approximation

If the Fermi level is far away ($> 3kT$) from the band edges W_C and W_V (as is the case for doping concentration of $n_D << N_C$ and $n_A << N_V$), then *Boltzmann's approximation* holds:

$$f(x) = \frac{1}{1+\exp\ x} \overset{x\gg1}{\cong} \frac{1}{\exp\ x} \quad \Longrightarrow \quad \begin{aligned} f_V(W) &= \exp\left(-\frac{W_F - W}{kT}\right) \\ f_L(W) &= \exp\left(-\frac{W - W_F}{kT}\right) \end{aligned}$$

Solving the integrals for the carrier concentrations with Boltzmann's approximation gives:

$$n_T = N_C \exp\left(-\frac{W_C - W_F}{kT}\right) \quad \text{with} \quad N_C = 2\left(\frac{2\pi m_n kT}{h^2}\right)^{3/2}$$

$$p = N_V \exp\left(-\frac{W_F - W_V}{kT}\right) \quad \text{with} \quad N_V = 2\left(\frac{2\pi m_p kT}{h^2}\right)^{3/2}$$

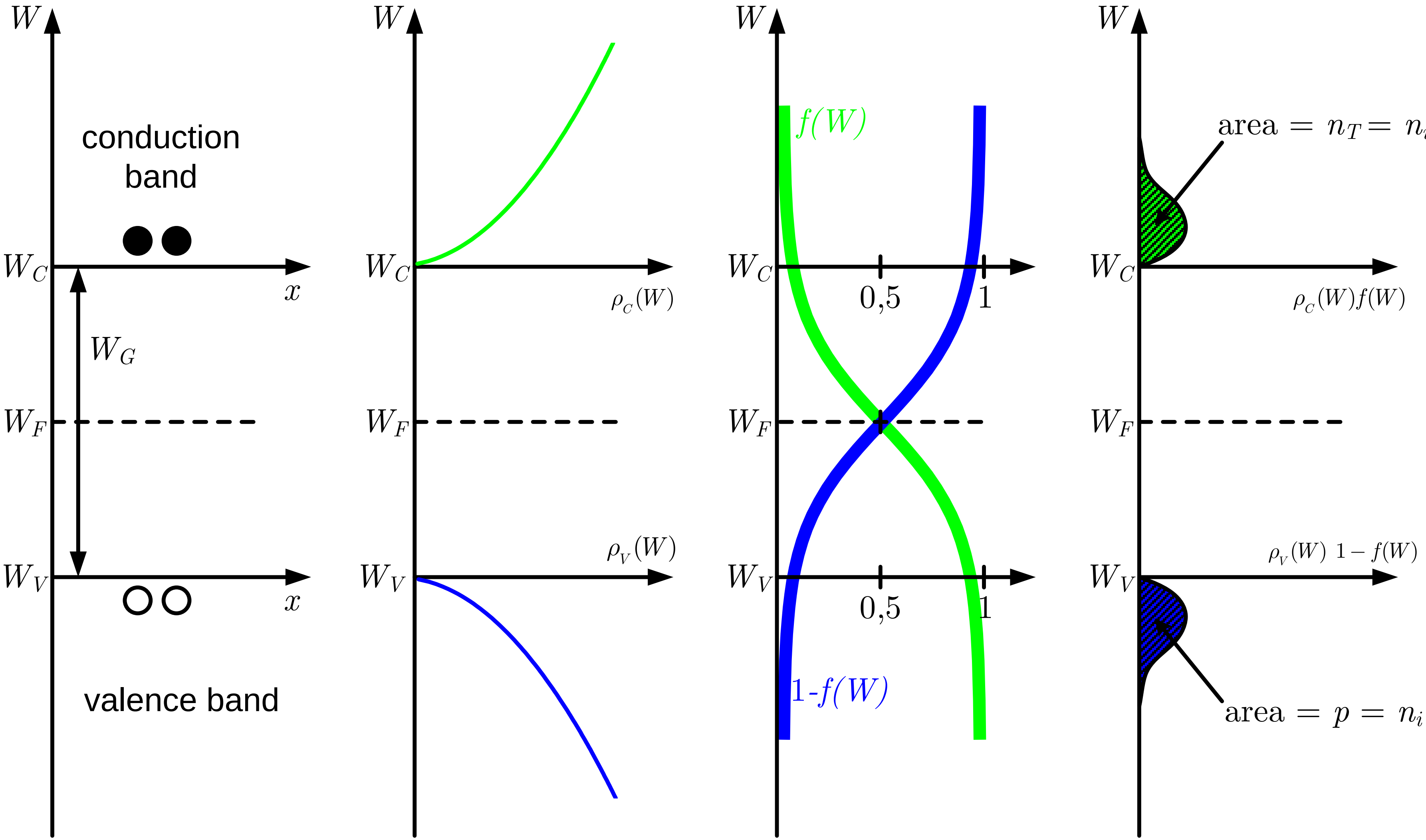
$$N_{C,V} \approx 10^{19}\,\text{cm}^{-3}$$

N_C and N_V are called effective density of states. Within kT from the band-edge, there are $0.75 N_{C,V}$ states. For intrinsic semiconductors follows:

$$n_i = \sqrt{n_T p} = \sqrt{N_C N_V} \exp\left(-\frac{W_G}{2kT}\right) \quad \text{and} \quad W_F = \frac{W_C + W_V}{2} + \frac{kT}{2} \ln\frac{N_V}{N_C}$$

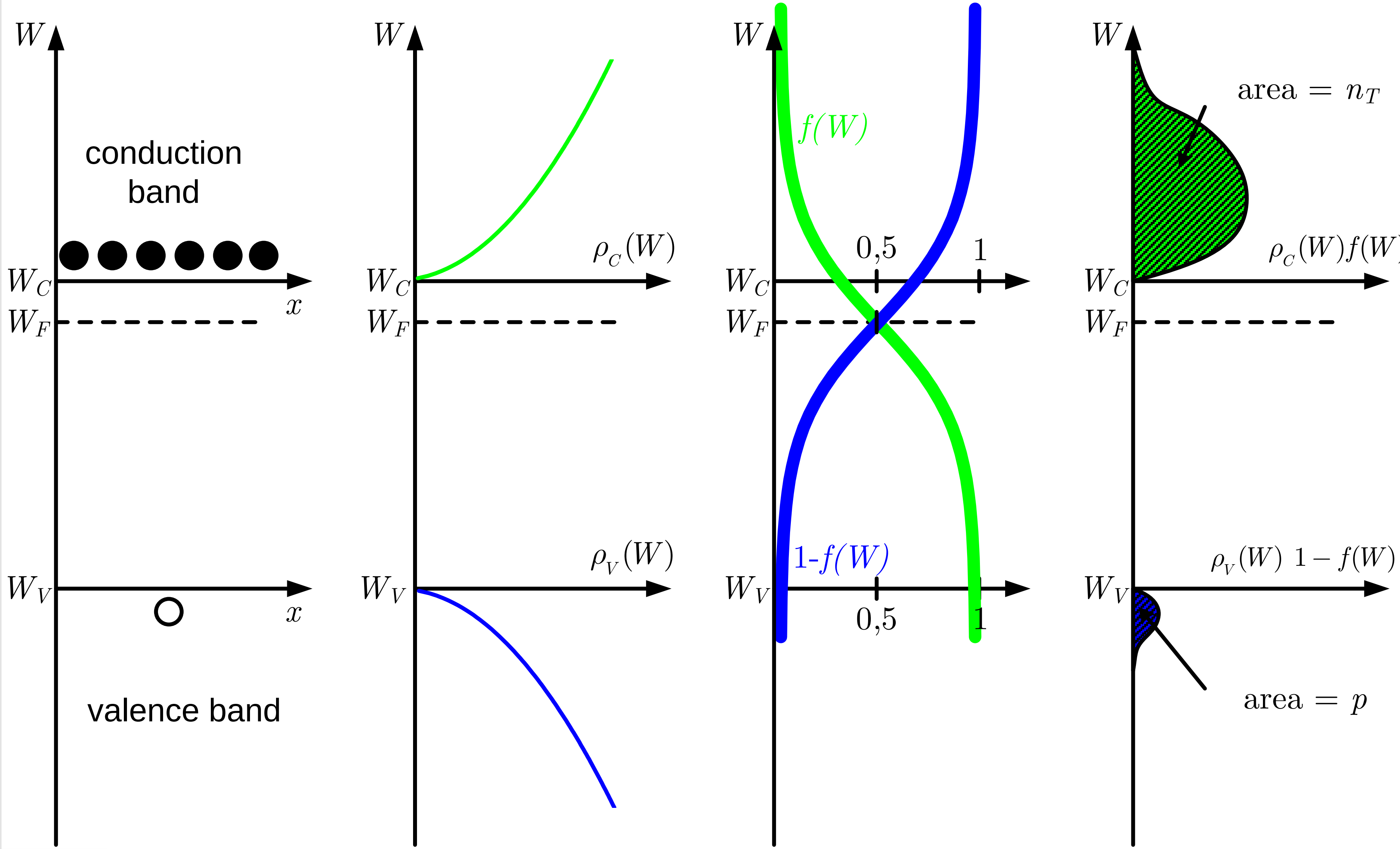
Visual Summary of Carrier Concentrations (1)

Intrinsic semiconductor in thermal equilibrium.



Visual Summary of Carrier Concentrations (2)

Extrinsic semiconductor (n-type) in thermal equilibrium.



Currents in Semiconductors

Drift current due to an electric field E :

$$\vec{J}_F = \vec{J}_{n,F} + \vec{J}_{p,F} = \; en_T\mu_n + ep\mu_p \;\; \vec{E} = \sigma\vec{E}$$

$\mu_{n,p}$	carrier mobility
e	elementary charge
σ	conductivity

Diffusion current due to a gradient of carrier concentration:

$$\vec{J}_D = \vec{J}_{n,D} + \vec{J}_{p,D} = eD_n \operatorname{grad} n_T - eD_p \operatorname{grad} p$$

Diffusion coefficients D_n and D_p for electrons and holes:

$$D_n = \mu_n U_T = \mu_n \frac{kT}{e} \quad \text{and} \quad D_p = \mu_p U_T = \mu_p \frac{kT}{e}$$

$U_T = kT/e$ is called temperature voltage (= 25 mV @ T = 293 K)

Diffusion lengths L_n and L_p for electrons and holes:

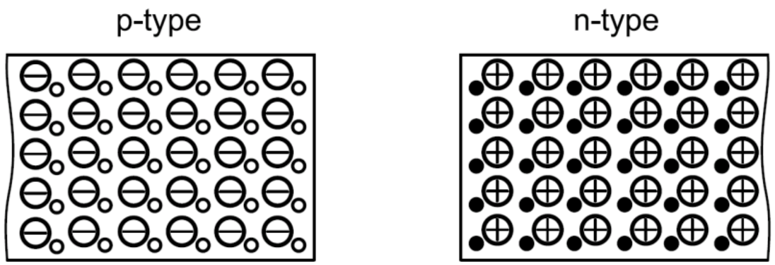
$$L_n = \sqrt{D_n \tau_n} \quad \text{and} \quad L_p = \sqrt{D_p \tau_p}$$

τ_n, τ_p are the minority carrier lifetimes of electrons and holes.

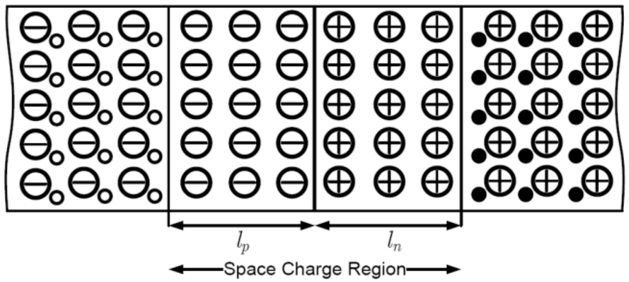
pn-junction in Thermal Equilibrium

Bringing together a p-type and n-type semiconductor.

Before contact:



After contact:



Electrons diffuse into the p-type semiconductor, and holes into the n-type semiconductor. The positively and negatively charged donor and acceptor ions in the *space charge region (SCR)* build up an electric field that counteracts diffusion.

In thermal equilibrium, there are zero net electron and hole currents, i.e. diffusion and drift currents compensate each other:

$$\vec{J}_p = \vec{J}_{p,F} + \vec{J}_{p,D} = 0 \qquad \vec{J}_n = \vec{J}_{n,F} + \vec{J}_{n,D} = 0$$

The *built-in potential* U_D of the pn-junction is given by:

$$U_D = U_T \ln \frac{n_D n_A}{n_i^2} = \frac{kT}{e} \ln \frac{n_D n_A}{n_i^2}$$

Current-Voltage Characteristics of pn-Diode

Applying an external voltage to the pn-junction → no equilibrium

Under *reverse bias* condition ($U < 0$, "+" at n-type, "-" at p-type), charge carriers are removed to increase the SCR width: $n_T p < n_i^2$

Under *forward bias* condition ($U > 0$, "+" at p-type, "-" at n-type), charge carriers are injected to reduce the SCR width: $n_T p > n_i^2$

Concentration of minority charge carriers at the edges of the SCR increase/decrease exponentially with the applied voltage U.

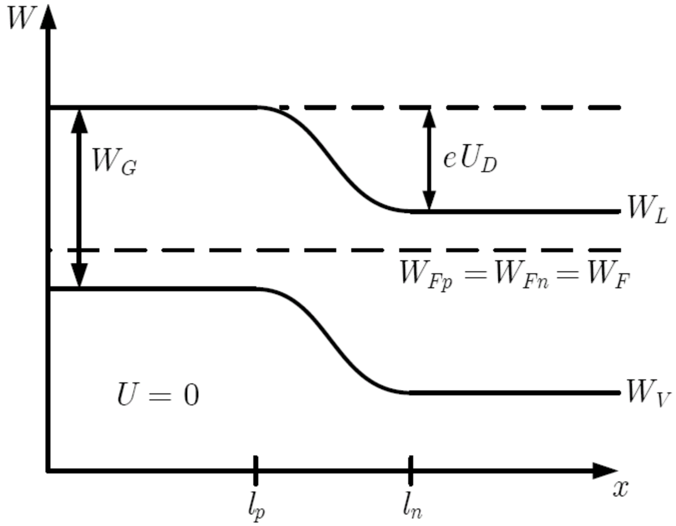
For example, the hole concentration change in the n-type region is:

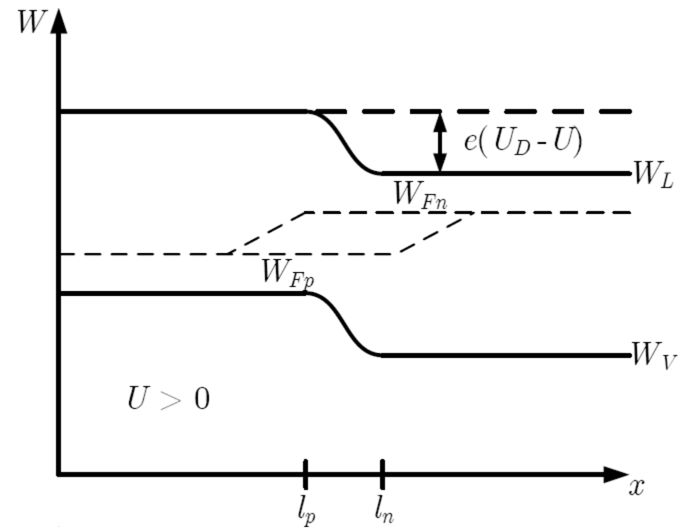
$$\Delta p_n(l_n) = p_{n0}\left(\exp\left(\frac{U}{U_T}\right) - 1\right) \qquad \Delta p_n(x) = p_{n0}(l_n)\exp\left(-\frac{x - l_n}{L_p}\right)$$

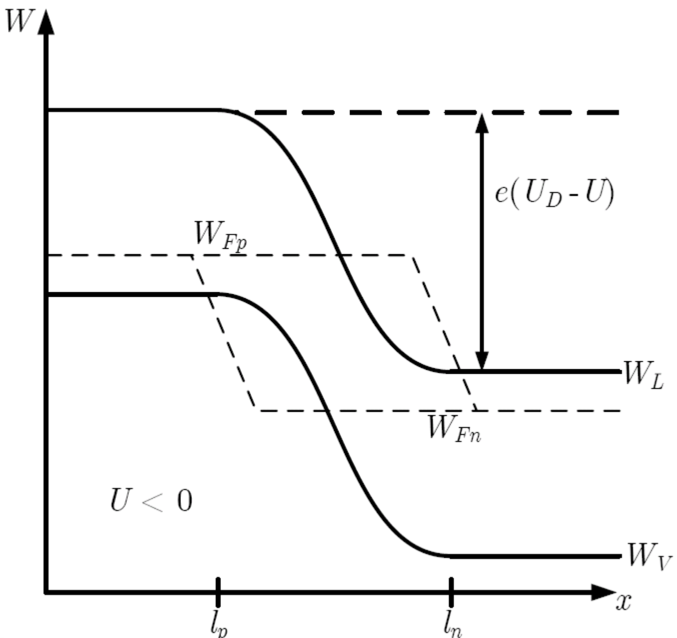
Assuming only diffusion currents outside the SCR, the current-voltage characteristics of the pn-diode follows as:

$$I = \underbrace{F\left(\frac{eD_n}{L_n} n_{p0} + \frac{eD_p}{L_p} p_{n0}\right)}_{\text{Saturation current } I_S}\left(\exp\left(\frac{eU}{kT}\right) - 1\right) = I_S\left(\exp\left(\frac{U}{U_T}\right) - 1\right)$$

Band Diagrams







In thermal equilibrium, the Fermi level is flat, i.e. no net current flows.

In non-equilibrium, the Fermi level splits up into the quasi Fermi levels (QFL) W_{Fn} and W_{Fp}. A gradient of the QFL indicates current flow.

$U > 0$ reduces barrier for carriers

$U < 0$ increases barrier for carriers

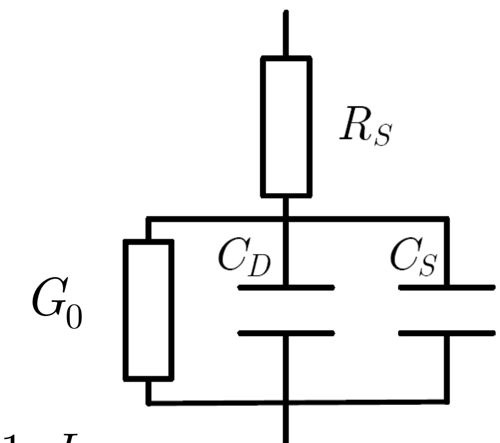
Depletion-Layer and Diffusion Capacitance

Depletion-layer capacitance (dominating if reverse biased): The total width of the SCR varies according to the applied voltage. Thus also the amount of charges in the SCR changes. Taking the formula for a parallel plate capacitor yields:

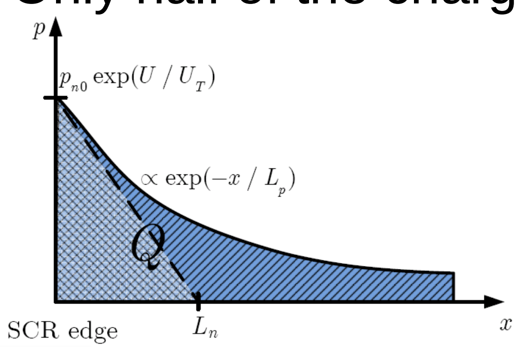
$$C_S = \varepsilon_0 \varepsilon_r \frac{F}{w(U)} \quad \text{with} \quad w(U) = l_n - l_p = \sqrt{\frac{2\varepsilon_0\varepsilon_r}{e} \frac{n_D + n_A}{n_D n_A} U_D - U}$$

Diffusion capacitance (dominating if forward biased): When applying a small AC signal, not all the minority carriers at the edge of the SCR follow the signal instantaneously. The stored minority charge (here: holes in an n-type semiconductor) is given by:

$$Q = eF \int_0^\infty p_{n0} \exp\left(\frac{U}{U_T}\right) \exp\left(-\frac{x}{L_p}\right) \mathrm{d}x = eF L_p p_{n0} \exp\left(\frac{U}{U_T}\right)$$



Only half of the charge is modulated effectively:



$$Q = I\tau \qquad G_0 = \frac{\delta I}{\delta U} = \frac{I}{U_T}$$

$$C_D = \frac{1}{2}\frac{\delta Q}{\delta U} = \frac{1}{2}\frac{\delta I}{\delta U}\frac{\delta Q}{\delta I} = \frac{1}{2} G_0 \tau = \frac{1}{2}\frac{I}{U_T}\tau$$